

Bulk emission by higher-dimensional black holes: almost perfect blackbody radiation

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We study the Hawking radiation emitted into the bulk by $(D + 1)$ -dimensional Schwarzschild black holes. It is well-known that the black-hole spectrum departs from exact blackbody form due to the frequency dependence of the ‘greybody’ factors. For intermediate values of D ($3 \leq D \lesssim 10$), these frequency-dependent factors may significantly modify the spectrum of the emitted radiation. However, we point out that for $D \gg 1$, the typical wavelengths in the black-hole spectrum are much *shorter* than the size of the black hole. In this regime, the greybody factors are well described by the geometric-optics approximation according to which they are almost frequency-independent. Following this observation, we argue that for higher-dimensional black holes with $D \gg 1$, the total power emitted into the bulk should be well approximated by the analytical formula for perfect blackbody radiation. We test the validity of this analytical prediction with numerical computations.

I. INTRODUCTION

Models with large extra dimensions are widely regarded as the most promising candidates for a consistent unified theory of the fundamental forces [1, 2]. In these models it is usually assumed that the standard model fields are confined to a four-dimensional hypersurface (known as the ‘brane’), but gravity (and possibly scalar fields) is free to propagate in a higher-dimensional compact space (the ‘bulk’).

Such higher-dimensional models are intriguing because they suggest an elegant resolution for the so-called hierarchy problem. In particular, they may explain why gravity is perceived to be much weaker than the other forces. According to these higher-dimensional models, the traditional Planck scale, M_P , is only an effective energy scale derived from the fundamental higher-dimensional one, M_* , through the relation [1–4]

$$M_P^2 \sim M_*^{2+n} R^n, \quad (1)$$

where R and n are the size and number of extra dimensions, respectively. From (1) one deduces that if the volume of the compact space, $V \sim R^n$, is large (i.e if $R \gg \ell_P$, where $\ell_P \approx 10^{-35}m$ is the traditional Planck-length), then the $(4 + n)$ -dimensional Planck mass, M_* , will be much lower than the 4-dimensional one, M_P . Remarkably, by lowering the Planck scale M_* closer to the energy scale of modern accelerators, the possibility of producing miniature black holes during high-energy scattering processes now becomes more realistic [3, 5].

If the horizon of the formed black hole is much smaller than the size of the extra dimensions, $r_H \ll R$, then the produced black hole may be considered as a higher-dimensional object that is submerged into the extra-dimensional spacetime [3]. If created, these mini black holes are expected to evaporate quickly by the emission of thermal Hawking radiation [6]. It is hoped that this characteristic radiation could be detected in future high-energy experiments. If detected, this radiation may pro-

vide an experimental verification of the celebrated Hawking evaporation process.

A higher-dimensional black hole emits radiation both in the bulk and on the brane. It is usually assumed that only gravitons (and possibly scalar fields) can propagate in the bulk. Thus, these are the only types of fields allowed to be emitted in the bulk during the Hawking evaporation phase [3]. It is important to realize that for an observer located on the brane, the radiation emitted in the bulk will be perceived as a missing energy signal. On the other hand, radiation on the brane may be detected directly. Nevertheless, in order to have a complete picture of the characteristics of the radiation spectrum on the brane, it is important to know how much energy is emitted (lost) in the bulk [3].

The non-trivial spacetime exterior to the black-hole horizon is characterized by an effective scattering potential. This potential barrier scatters part of the outgoing radiation back into the black hole [3]. As a consequence of this radiation backscattering, the power spectrum that would be detected by an observer at spatial infinity would not be universal. In particular, it would depend on several parameters: the energy ω of the emitted particle, its spin s , and the dimensionality $(D + 1)$ of spacetime [3] (We denote by $D = 3 + n$ the total number of spatial dimensions). The dependence of the emission spectrum on all these parameters is encoded into the ‘greybody factor’ $\sigma(\omega)_{sD}$. This factor acts as a filtering function which characterizes the interaction of the emitted quanta with the curvature scattering potential which surrounds the black hole. This interaction modifies the thermal radiation spectrum [3] [see Eq. (7) below.]

The greybody factors can be calculated analytically in the low-energy $\omega r_H \ll 1$ and high-energy $\omega r_H \gg 1$ regimes [3, 7]. However, for moderate values of D most of the Hawking radiation is actually emitted around $\omega r_H \approx 1$, where the analytical approximations break-down. Thus, *numerical* integration of the perturbed field equations seems necessary in order to compute the exact greybody factors and to find the corresponding black-hole

emission power [3, 4, 8–10].

Nevertheless, in this paper we point out that for higher-dimensional spacetimes with $D \gg 1$, the typical wavelengths emitted into the bulk are much *shorter* than the size of the black hole. In this regime, the greybody factors are well described by the geometric-optics approximation. As a consequence, we shall show below that for higher-dimensional black holes with $D \gg 1$, the total power emitted into the bulk is well approximated by the *analytical* formula for perfect (undistorted) blackbody radiation.

II. HAWKING RADIATION IN THE BULK

We consider higher-dimensional black holes that have horizon radius much smaller than the size of the extra dimensions, $r_H \ll R$. These mini black holes are completely submerged into a $(D+1)$ -dimensional spacetime that, to a very good approximation, has one timelike and D non-compact spacelike coordinates [3]. If we further assume that the black hole is spherically-symmetric with ADM mass \mathcal{M} , the spacetime outside the horizon is described by the $(D+1)$ -dimensional Schwarzschild-Tangherlini metric [11, 12] (we use natural units in which $G = c = 1$):

$$ds^2 = -H(r)dt^2 + H(r)^{-1}dr^2 + r^2 d\Omega^{(D-1)}, \quad (2)$$

where

$$H(r) = 1 - \left(\frac{r_H}{r}\right)^{D-2}. \quad (3)$$

Here

$$r_H = \left[\frac{16\pi\mathcal{M}}{(D-1)A_{D-1}} \right]^{\frac{1}{D-2}} \quad (4)$$

is the black hole's radius and

$$A_{D-1} = \frac{2\pi^{D/2}}{\Gamma(D/2)} \quad (5)$$

is the area of a unit $(D-1)$ -sphere (The black hole's area is given by $A_H = A_{D-1}r_H^{D-1}$.) The Hawking temperature of the black hole is given by [3]

$$T = \frac{(D-2)\hbar}{4\pi r_H}. \quad (6)$$

For one helicity degree of freedom, the energy emitted per unit time into the bulk by a $(D+1)$ -dimensional black hole is given by [3]:

$$P_D = \sum_j \int_0^\infty \sigma(\omega)_{sjD} \frac{\hbar\omega dV_D(\omega)}{(e^{\hbar\omega/T} - 1)(2\pi)^D}, \quad (7)$$

where s is the spin of the emitted quantum ($s = 2$ for gravitons and $s = 0$ for scalars), j its angular momentum

quantum number, $\sigma(\omega)_{sjD}$ is the frequency-dependent greybody factor of the spacetime, and

$$dV_D(\omega) = [2\pi^{D/2}/\Gamma(D/2)]\omega^{D-1}d\omega \quad (8)$$

is the volume in frequency space of the shell $(\omega, \omega + d\omega)$. Substituting (8) into (7), one obtains

$$P_D = \frac{2\pi^{D/2}}{(2\pi)^D \Gamma(D/2)} \sum_j \int_0^\infty \sigma(\omega)_{sjD} \frac{\hbar\omega^D d\omega}{(e^{\hbar\omega/T} - 1)} \quad (9)$$

for the power radiated into the bulk per one degree of freedom by the $(D+1)$ -dimensional black hole.

As discussed above, the factor $\sigma(\omega)_{sjD}$ is not universal – it has a complicated dependence on several parameters of the system: the energy ω of the emitted particle, its spin s , its angular momentum j , and the number D of spatial dimensions. Thus, this factor modifies the radiation spectrum that reaches an observer at spatial infinity. In particular, such an observer would not detect a perfect thermal radiation. The curvature potential barrier which surrounds the black hole mostly blocks the low energy part ($\omega r_H \ll 1$) of the emission spectrum (this should be contrasted with pure black body radiation in a flat spacetime). As a consequence, the black-hole power spectrum is expected to peak at higher frequencies as compared to those of perfect blackbody radiation with the same temperature.

We point out that the distribution $\omega^D/(e^{\hbar\omega/T} - 1)$ in Eq. (9) peaks at the characteristic frequency

$$\omega^* = \frac{DT}{\hbar} [1 - e^{-D} + O(e^{-2D})]. \quad (10)$$

Taking cognizance of Eq. (6) for the black hole's temperature, one finds for $D \gg 1$

$$\omega^* r_H = \frac{D(D-2)}{4\pi} \gg 1. \quad (11)$$

This implies that for higher-dimensional black holes with $D \gg 1$, the typical wavelengths in the Hawking radiation are much *shorter* than the size of the black hole. Since for $D \gg 1$ the integral in (9) is dominated by large frequencies around ω^* , one may approximate P_D by

$$P_D \simeq \frac{2\pi^{D/2}}{(2\pi)^D \Gamma(D/2)} \sum_j \sigma(\omega^*)_{sjD} \int_0^\infty \frac{\hbar\omega^D d\omega}{(e^{\hbar\omega/T} - 1)}. \quad (12)$$

In the short wavelength regime $\omega r_H \gg 1$ (the geometrical optics limit), geometric arguments [3, 13–15] show that the absorption cross-section $\sigma_{\text{abs}} \equiv \sum_j \sigma(\omega)_{sjD}$ is a constant *independent* of ω and s [3]: Consider a massless particle in a circular orbit around a black hole described by the line-element (2). Its equation of motion $p^\mu p_\mu = 0$ takes the form [3]

$$\left(\frac{1}{r} \frac{dr}{d\phi}\right)^2 = \frac{1}{b^2} - \frac{H(r)}{r^2}, \quad (13)$$

where b is the ratio of the angular momentum of the particle over its linear momentum. Since the left-hand-side of (13) is positive definite, the classically accessible regime of the particle is defined by the relation $b < \min(r/\sqrt{H})$. Thus, the closest distance the particle can get from the black hole is given by [3, 14]

$$b = r_c \equiv \left(\frac{D}{2}\right)^{\frac{1}{D-2}} \sqrt{\frac{D}{D-2}} r_H. \quad (14)$$

The radius r_c defines the absorptive area of the black hole at high energies. For large values of the energy of the scattered particle, the greybody factor σ_{abs} becomes equal to the area of an absorptive body of radius r_c which is projected on a plane parallel to the orbit of the moving particle [3, 13]:

$$\sigma_{\text{abs}} = \frac{2\pi}{D-1} \frac{\pi^{\frac{D-3}{2}}}{\Gamma(\frac{D-1}{2})} r_c^{D-1}. \quad (15)$$

Note that σ_{abs} can also be written as

$$\sigma_{\text{abs}} = \frac{1}{\sqrt{\pi}(D-1)} \frac{\Gamma(\frac{D}{2})}{\Gamma(\frac{D-1}{2})} \left(\frac{r_c}{r_H}\right)^{D-1} A_H. \quad (16)$$

In [3, 16] it was demonstrated by explicit numerical computations that both the total absorption cross-section of gravitational perturbations (composed of tensor, vector, and scalar type perturbations [7]) and the absorption cross-section of scalar fields tend to the classical expression (16) in the high-energy regime $\omega r_H \gg 1$.

Substituting (6) and (16) into (12), and using the relation

$$\int_0^\infty \frac{x^D dx}{e^x - 1} = \zeta(D+1) \Gamma(D+1), \quad (17)$$

where $\zeta(z)$ is the Riemann zeta function, one finds

$$P_D^{\text{tot}} \simeq N_D \left(\frac{D-2}{4\pi}\right)^{D+1} \left(\frac{r_c}{r_H}\right)^{D-1} \frac{D\zeta(D+1)\hbar}{\pi r_H^2} \quad (18)$$

for the total power radiated into the bulk from a $(D+1)$ -dimensional black hole with $D \gg 1$, where N_D is the effective number of massless degrees of freedom (the number of polarization states). Massless scalars contribute 1 to N_D , while gravitational waves contribute $(D+1)(D-2)/2$ to N_D [7].

Note that for $D \gg 1$ one has $(r_c/r_H)^{D-1} \rightarrow De/2$. Thus, the total radiated power (18) can be approximated by the compact formula

$$P_D^{\text{tot}} \simeq \frac{8\pi N_D \hbar}{e r_H^2} \left(\frac{D}{4\pi}\right)^{D+3}. \quad (19)$$

III. ANALYTICAL VS. NUMERICAL RESULTS

It is of interest to test the validity of the approximated analytical formula (18) for the energy emission rate into

the bulk from a $(D+1)$ -dimensional Schwarzschild black hole. Our analytical treatment is based on the observation (11) according to which the typical wavelengths in the Hawking radiation are much *shorter* than the size of the black hole in the $D \gg 1$ regime: $\omega^* r_H = \frac{D(D-2)}{4\pi} \gg 1$. In this regime, the geometric-optics approximation predicts a frequency-*independent* greybody factors given by Eq. (16). This implies that the dominant part of the black-hole emission spectrum (around ω^*) is hardly affected by the curvature potential barrier in the $D \gg 1$ regime. For this reason, we expect the black-hole emission properties (in the $D \gg 1$ regime) to be well approximated by the emission properties of a perfect blackbody with the same temperature.

We shall first compare the location of the peak in the black-hole emission spectrum with the value predicted by the blackbody analytical expression (10). In Table I we display the ratio $\Omega_D \equiv \frac{\omega_{\text{blackhole}}^*}{\omega_{\text{blackbody}}^*}$ between the numerically computed [3, 4, 8–10] peak-frequency and the analytical prediction (10). We present results for the total gravitational spectrum and for the scalar spectrum in the bulk. As explained above, the curvature potential barrier outside the black hole mostly affect the low-frequency part of the emission spectrum. The result is that the black-hole power spectrum is expected to peak at higher frequencies as compared to those of perfect blackbody radiation with the same temperature. From Table I one indeed finds $\omega_{\text{blackhole}}^* > \omega_{\text{blackbody}}^*$.

For scalar waves the agreement $\omega_{\text{blackhole}}^* \simeq \omega_{\text{blackbody}}^*$ ($\Omega_D \simeq 1$) is quite impressive already at $D = 3$. This indicates that the dominant part of the scalar emission spectrum (around ω^*) is hardly affected by the curvature potential barrier for all D values. We therefore expect the scalar emission power to follow closely the blackbody analytical expression (18) for all D values. Below we shall confirm this expectation.

For gravitational waves one finds a large deviation between the peak values $\omega_{\text{blackhole}}^*$ and $\omega_{\text{blackbody}}^*$ in the case of three spatial dimensions. We therefore expect the gravitational emission rate to be suppressed as compared to the blackbody analytical prediction (18) for $D = 3$. Below we shall confirm this expectation. However, for $D = 10$ one finds a good agreement between $\omega_{\text{blackhole}}^*$ and $\omega_{\text{blackbody}}^*$. This indicates that for $D \gg 1$, the dominant part of the gravitational emission spectrum is hardly affected by the curvature potential barrier.

We shall now compare the total power emitted into the bulk from a $(D+1)$ -dimensional black hole with the power predicted by the blackbody analytical model. In Table II we display the ratio $\Pi_D \equiv \frac{P_{\text{blackhole}}}{P_{\text{blackbody}}}$ between the numerically computed [3, 4, 8–10] black-hole emission power and the blackbody analytical expression (18). We present results for the total emission of gravitational waves and scalar waves in the bulk.

For scalar waves, the agreement between the black-hole numerical results and the blackbody analytical prediction is quite impressive already at $D = 3$. This confirms our

D	$\Omega_D(\text{gravitational})$	$\Omega_D(\text{scalar})$
3	1.590	1.058
10	1.028	1.017

TABLE I: The ratio $\Omega_D \equiv \omega_{\text{blackhole}}^*/\omega_{\text{blackbody}}^*$ between the numerically computed peak-frequency and the approximated analytical prediction (10). We present results for the total gravitational spectrum and for the scalar spectrum in the bulk. Due to the presence of the curvature potential barrier surrounding the black hole, one expects to find $\Omega_D > 1$ with $\Omega_D \rightarrow 1$ for $D \gg 1$.

earlier expectation. For gravitational waves, the black-hole emission power for $D = 3$ is suppressed as compared to the blackbody analytical prediction (18). Again, this confirms our earlier expectation. However, one learns from Table II that the agreement between the black-hole numerical data and the blackbody analytical formula (18) improves considerably as the number D of spatial dimensions increases. This is to be expected, since the larger is the number of spatial dimensions, the larger are the characteristic frequencies which dominate the emission spectrum, see Eq. (11). Thus, the larger is the number of spatial dimensions, the smaller is the relative part of the emission spectrum which is blocked by the curvature potential barrier which surrounds the black hole.

D	$\Pi_D(\text{gravitational})$	$\Pi_D(\text{scalar})$
3	0.027	1.037
4	0.247	0.974
5	0.543	0.945
6	0.715	0.934
7	0.783	0.931
8	0.814	0.929
9	0.833	0.922
10	0.846	0.928

TABLE II: The ratio $\Pi_D \equiv P_{\text{blackhole}}/P_{\text{blackbody}}$ between the numerically computed black-hole emission power and the blackbody analytical expression (18). We present results for the total emission of gravitational waves and scalar waves in the bulk. One expects to find $\Pi_D \rightarrow 1$ for $D \gg 1$.

IV. SUMMARY

We have studied the Hawking radiation emitted into the bulk by $(D + 1)$ -dimensional Schwarzschild black

holes. It is well-known that the black-hole spectrum departs from exact blackbody form. The emission spectrum is influenced by the frequency dependence of the greybody factors [see Eq. (9)]. For the canonical case of three spatial dimensions, the frequency-dependent greybody factors must be computed *numerically* in order to obtain the exact black-hole emission power [9, 15]. This is also the situation for intermediate values of D [3, 4, 8].

However, in this paper we have pointed out that for $D \gg 1$, the typical wavelengths in the bulk spectrum are much *shorter* than the size of the black hole. In this regime, the greybody factors are well described by the geometric-optics approximation. According to this approximation, the greybody factors are frequency-independent; they are simply given by the projected area of an absorptive body of radius r_c . This implies that for higher-dimensional Schwarzschild black holes with $D \gg 1$, the total power emitted into the bulk is well described by the analytical formula (18) of perfect blackbody radiation. We have tested this prediction and found a reasonably good agreement already at $D \simeq 10$ between the (numerically computed) black-hole power and the (analytically calculated) blackbody power.

As emphasized in [7], the relative contributions of the higher partial waves to the emission power increase with D . Thus, contributions from high values of l are needed in order to obtain accurate *numerical* results for large values of D . For example, in four dimensions the contribution of the $l = 2$ mode is two orders of magnitude larger than the contribution of the $l = 3$ mode. However, in ten dimensions the first 10 modes must be considered for a meaningful numerical result (see [7]). Therefore, precise numerical values for very large number of spatial dimensions require the most CPU-time. This fact limits the utility of the numerical computations to moderate values of D . In fact, the numerical results that appear in the literature are limited to the case $D \leq 10$. Luckily, for higher values of D the agreement between the analytical approximation (18) and the exact results is expected to be very good. Thus, an analytical formula like (18) allows the calculation of the emission power in cases which would otherwise require long numerical integration times.

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